STA 711 Homework 9

Due: Monday, April 10, 12:00pm (noon) on Canvas.

Instructions: Submit your work as a single PDF. For this assignment, you may include written work by scanning it and incorporating it into the PDF. Include all R code needed to reproduce your results in your submission.

1. In class, we have worked with Wald confidence intervals for a binomial proportion. Now let's try inverting the test. Suppose we have data $X_1, ..., X_n \overset{iid}{\sim} Bernoulli(p)$. Derive a $1 - \alpha$ confidence interval for p by inverting the LRT of $H_0 : p = p_0$ vs. $H_A : p \neq p_0$. (It may be difficult to completely simplify the interval).

2. Suppose $X_1, ..., X_n \overset{iid}{\sim} N(\theta, \theta)$, where $\theta > 0$. Find a pivotal quantity $Q(X_1, ..., X_n, \theta)$, and use the quantity to create a $1 - \alpha$ confidence interval for θ.

3. Suppose $X_1, ..., X_n \overset{iid}{\sim} Uniform[\theta - \frac{1}{2}, \theta + \frac{1}{2}]$. Find a $1 - \alpha$ confidence interval for θ.

4. Suppose that $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

   (a) If σ^2 is known, the interval for μ is $\overline{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$, and the *width* of the interval is $2z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$. Find the minimum value of n so that a 95% confidence interval for μ will have a length of at most $\sigma/4$.

   (b) If σ^2 is unknown, the interval for μ is $\overline{X} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}$, where $s^2 = \frac{1}{n-1} \sum\limits_{i=1}^{n} (X_i - \overline{X})^2$. Find the minimum value of n such that, with probability 0.9, a 95% confidence interval for μ will have a length of at most $\sigma/4$.

5. Suppose that $X_1, ..., X_n \overset{iid}{\sim} Poisson(\lambda)$. A $1 - \alpha$ Wald interval for λ is $\widehat{\lambda} \pm z_{\alpha/2} \sqrt{\frac{\widehat{\lambda}}{n}}$, where $\widehat{\lambda} = \overline{X}$. Clearly, the variance of $\widehat{\lambda}$ depends on λ.

   (a) Find a variance stabilizing transformation g such that the variance of $g(\widehat{\lambda})$ does not depend on λ.

   (b) Use (a) to find a $1 - \alpha$ confidence interval for λ.