STA 711 Homework 2

Due: Friday, January 27, 12:00pm (noon) on Canvas.

Instructions: Submit your work as a single PDF. For this assignment, you may include written work by scanning it and incorporating it into the PDF. Include all R code needed to reproduce your results in your submission.

Maximum likelihood estimation

1. Let $Y_1, ..., Y_n \overset{iid}{\sim} Poisson(\lambda)$, and let $\mathbf{Y} = (Y_1, ..., Y_n)$ denote the combined sample.

   (a) Write down the likelihood $L(\lambda|\mathbf{Y})$.

   (b) Find the maximum likelihood estimator $\widehat{\lambda}$ of λ.

2. Let $Y_1, ..., Y_n \overset{iid}{\sim} Exponential(\theta)$, so $f(y|\theta) = \theta e^{-\theta y}$.

   (a) Write down the likelihood $L(\theta|\mathbf{Y})$.

   (b) Find the maximum likelihood estimator $\widehat{\theta}$ of θ.

   (c) Show that $Y_{(1)} \sim Exponential(n\theta)$.

   (d) Suppose that instead of observing $Y_1, ..., Y_n$, we only observe the minimum $Y_{(1)}$. What would be the maximum likelihood estimator of θ?

3. Let $Y_1, ..., Y_n$ be iid from a distribution with pdf

$$f(y|\theta) = \theta y^{-2} \mathbb{1}\{y \geq \theta\},$$

   where $\theta > 0$. Find the maximum likelihood estimator of θ.

4. Let $Y_1, ..., Y_n$ be iid with one of two pdfs. If $\theta = 0$, then

$$f(y|\theta) = \begin{cases} 1 & 0 < y < 1 \\ 0 & \text{else.} \end{cases}$$

   If $\theta = 1$, then

$$f(y|\theta) = \begin{cases} \frac{1}{2\sqrt{y}} & 0 < y < 1 \\ 0 & \text{else.} \end{cases}$$

   Find the maximum likelihood estimator of θ.

5. Let Y be a single observation from a normal distribution with mean θ and variance θ^2, where $\theta > 0$. Find the maximum likelihood estimator of θ^2.

6. Let $Y_1, ..., Y_n$ be a random sample from a distribution with pdf

$$f(y|\mu, \sigma) = \frac{1}{\sigma} \exp\left\{-\left(\frac{y-\mu}{\sigma}\right)\right\} \mathbb{1}\{y \geq \mu\},$$

   where $-\infty < \mu < \infty$, and $\sigma > 0$.

   (a) Find the maximum likelihood estimators of μ and σ. *(Hint: find $\widehat{\mu}$ first)*

   (b) Let $\tau(\mu, \sigma) = \mathbb{P}_{\mu,\sigma}(Y_1 \geq t)$, where $t > \mu$, and $\mathbb{P}_{\mu,\sigma}$ denotes probability when μ, σ are the true parameters. Find the maximum likelihood estimator of $\tau(\mu, \sigma)$.